

# Symbolic model checking of multi-modal logics: uniform strategies and rich explanations

*(Model checking symbolique de logiques multi-modales :  
stratégies uniformes et explications riches)*

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Public defense

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## Running example: Mastermind



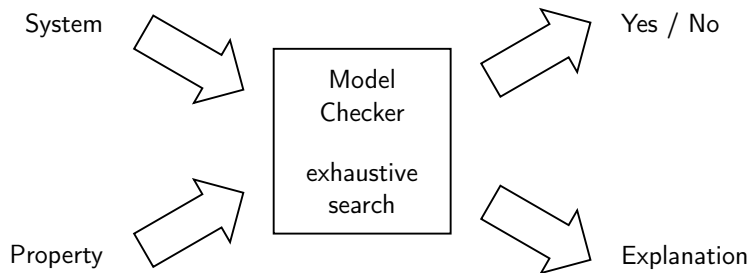
Simplified Mastermind: 3 colors, 3 turns, 2 pegs (different colors)

# Running example: Mastermind



Simplified Mastermind: 3 colors, 3 turns, 2 pegs (different colors)

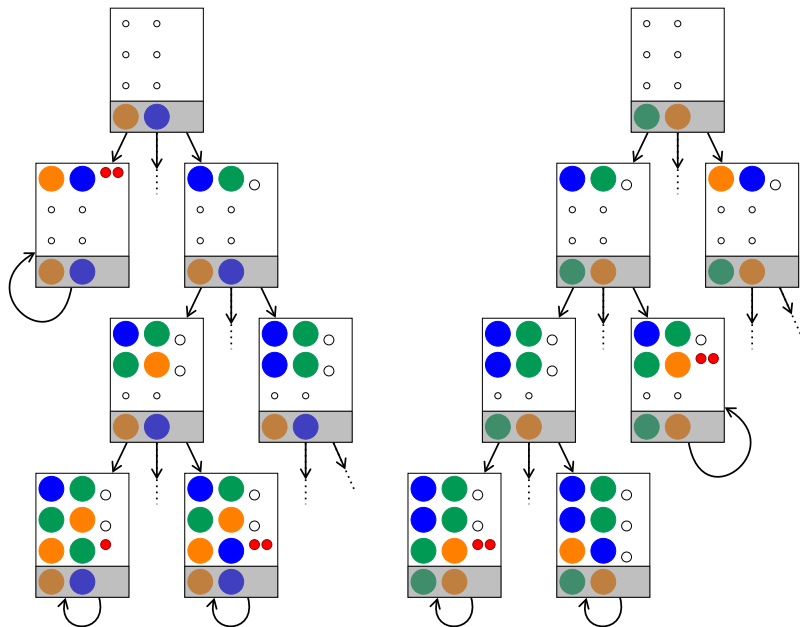
Model checking = a **verification** technique



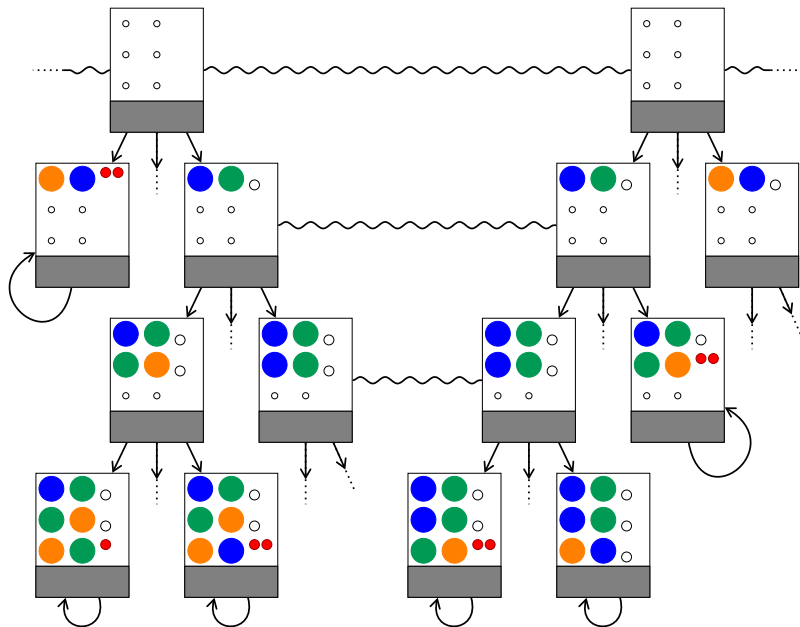
Other verif. techniques: testing, simulation, proof-based approaches



System modeled as a **finite-state machine** (1122 states)



# System modeled as a **finite-state machine**



# Properties expressed within a **logic**

⇒ The logic defines what properties can be expressed

⇒ The properties have a mathematical meaning:  
a **formal semantics**

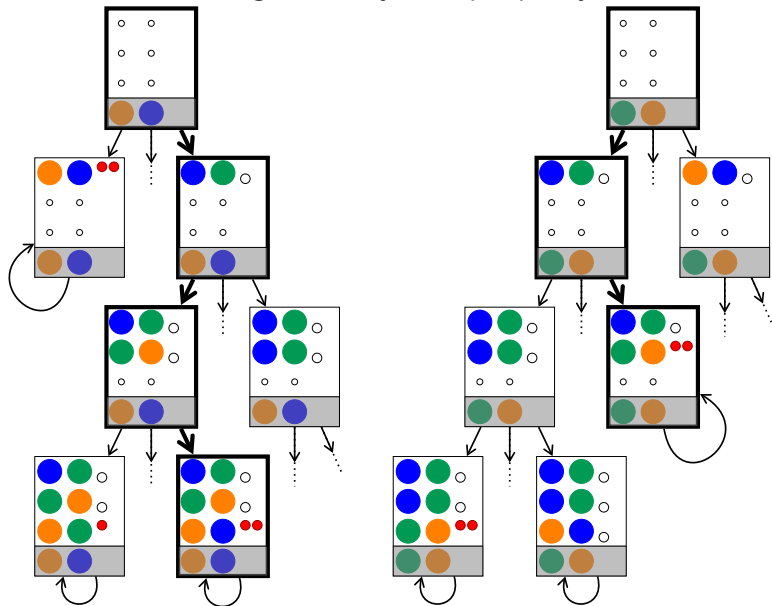
Example in temporal logic:

*"it is possible to win the game"*

**EF** *win*



Model checking = **exhaustive** search guided by the property



# Symbolic model checking



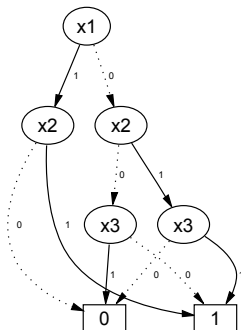
**state-space explosion** problem:

$T$  turns,  $C$  colors,  $P$  pegs  $\Rightarrow$  up to  $C^{P(T+1)}$  states  
standard Mastermind  $\Rightarrow 8^{4 \times (12+1)} \approx 9 \times 10^{46}$  states

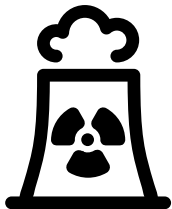
Solution:

use **Binary Decision Diagrams (BDDs)**  
to represent and manipulate

- the system
- the transitions
- sets of states



Applications: safety-critical systems



# Multi-modal logics

Reason about **several aspects** of the model:  
time, knowledge, strategies, etc.

*"the pilot **will eventually know** that he may land"*

**AF**  $K_{pilot}$  *authorized*

*"the doctor is **always aware** that the patient is not dead"*

**AG**  $K_{doctor}$   $\neg$ *dead*

*"the power plant controller knows he can avoid explosions"*

$K_{controller}$   $\llbracket controller \rrbracket$  **G**  $\neg$ *explosion*

# Thesis contributions

1. Model checking techniques for **uniform strategies**
2. A framework for multi-modal logic **rich explanations** generation and manipulation

# Outline

## **Model checking uniform strategies**

- Uniform strategies**

- Model checking approaches**

- Comparison with existing approaches**

- Conclusion on model checking uniform strategies**

## **Rich explanations for multi-modal logics**

## **Conclusion**

Mastermind: is there a **strategy** to win the game?



How can we find such a strategy?

# What is a strategy?

A (general) strategy =

what to do (what **action** to play) in each state



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Just play the solution

⇒ unrealistic because the player cannot see the solution

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Just play the solution  
⇒ unrealistic because the player cannot see the solution

A uniform strategy =  
what to do in each **observed** situation =  
same actions in **indistinguishable** states



cannot play the solution in the initial state  
because the player does not see it

$ATL_{ir}$ : a logic to reason about uniform strategies

Look for uniform strategies to achieve some **objective**

*"the player has a strategy to **eventually** win the game"*

$\langle\langle \text{player} \rangle\rangle \mathbf{F} \text{ win}$

*"the player has a strategy to **never** put a blue peg"*

$\langle\langle \text{player} \rangle\rangle \mathbf{G} \text{ no blue peg}$

*"the player has a strategy to play only blue pegs at the next turn"*

$\langle\langle \text{player} \rangle\rangle \mathbf{X} \text{ all blue}$

The same uniform strategy must be winning  
for **all states indistinguishable from the states of interest!**

# The problem

Checking that there exists a winning uniform strategy for a given objective

- is difficult ( $\Delta_2^P$ -complete =  $P^{NP}$ -complete)
- had no solution since recently

# Contributions

Techniques for checking the existence of winning uniform strategies:

1. a **naïve** approach
2. an improved approach based on **partial** strategies
3. another approach building winning strategies from target states

They **enumerate** and **check** every uniform strategy of the agents

+ a way to remove **surely losing choices**  
before enumerating the strategies

# Outline

## **Model checking uniform strategies**

Uniform strategies

## **Model checking approaches**

Comparison with existing approaches

Conclusion on model checking uniform strategies

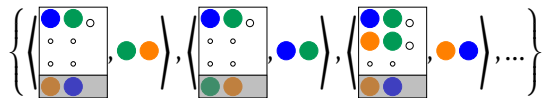
Rich explanations for multi-modal logics

Conclusion

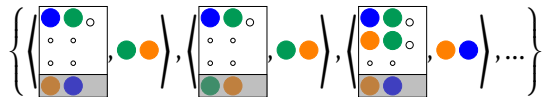
# Representing strategies

A strategy = what action to play in each state

⇒ represent a strategy as a set of **state-action pairs** (moves)



A uniform strategy is represented as a set of **non-conflicting** moves



⇒ can be easily represented using binary decision diagrams



# Checking one strategy for a given objective



Based on fixpoint computations, easy to compute (PTIME)

$Pre_{\langle\Gamma\rangle}(Q', f_\Gamma)$  = states in which  $\Gamma$  can enforce to reach a state in  $Q'$  by using actions provided by  $f_\Gamma$

$filter_{\langle\Gamma\rangle\mathbf{X}}(Q', f_\Gamma) = Pre_{\langle\Gamma\rangle}(Q', f_\Gamma)$   
= states in which  $\Gamma$  can enforce paths with second state in  $Q'$  by using actions in  $f_\Gamma$

$filter_{\langle\Gamma\rangle\mathbf{F}}(Q', f_\Gamma) = \mu Z. Q' \cup Pre_{\langle\Gamma\rangle}(Z, f_\Gamma)$   
= states in which  $\Gamma$  can enforce paths reaching  $Q'$  by using actions in  $f_\Gamma$

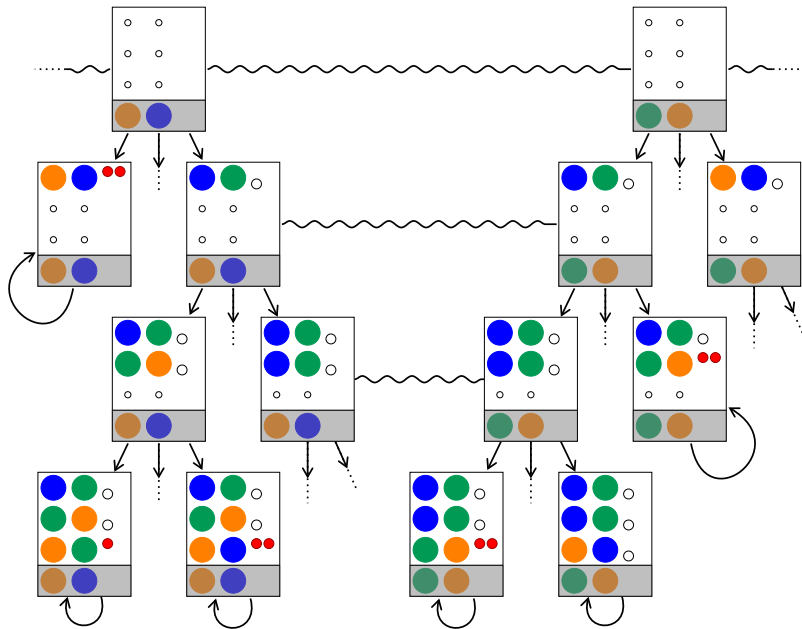
$filter_{\langle\Gamma\rangle\mathbf{G}}(Q', f_\Gamma) = \nu Z. Q' \cap Pre_{\langle\Gamma\rangle}(Z, f_\Gamma)$   
= states in which  $\Gamma$  can enforce paths staying in  $Q'$  forever by using actions in  $f_\Gamma$

# The naive approach

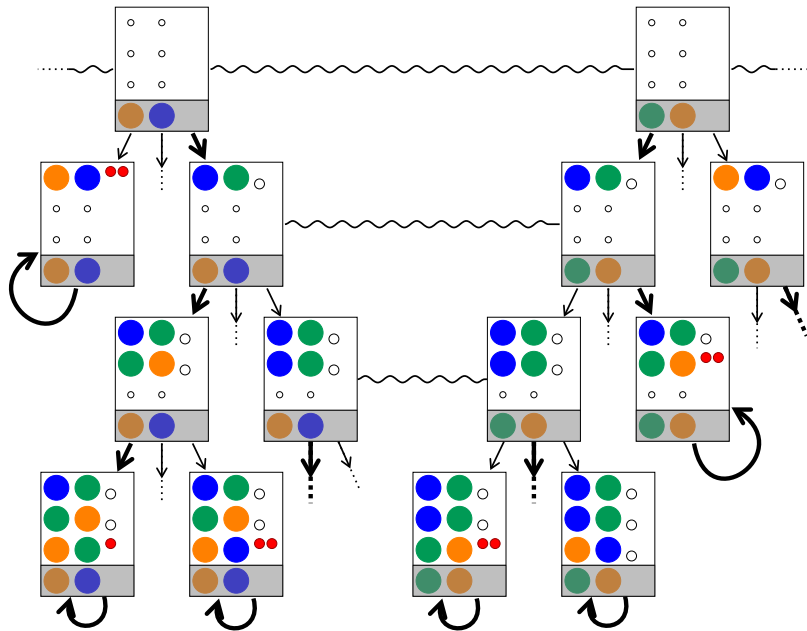
To compute the states for which there exists a winning uniform strategy for some objective

1. split the whole model into **uniform strategies**  $f_{\Gamma}$
2. compute the states for which the **strategy is winning** with the corresponding *filter* algorithm
3. keep the states for which the strategy is winning for **all indistinguishable states**

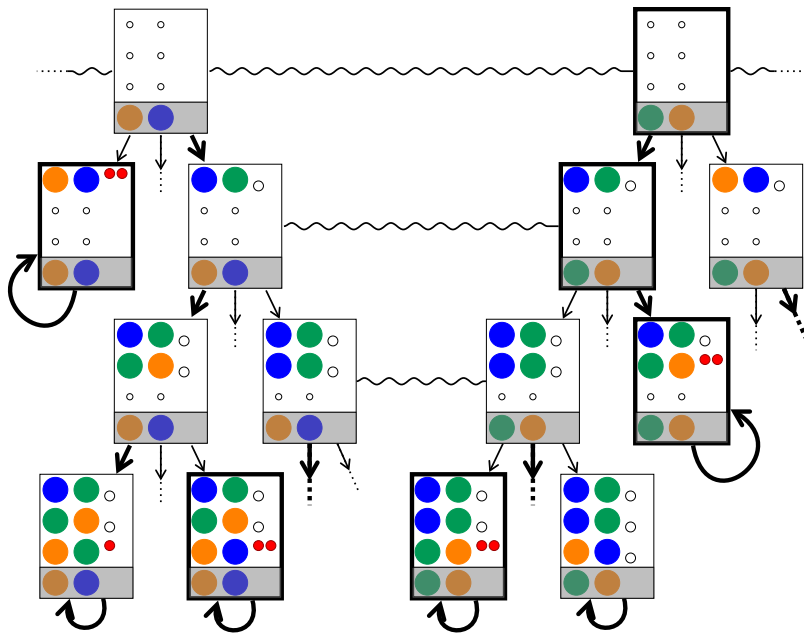
# The naive approach



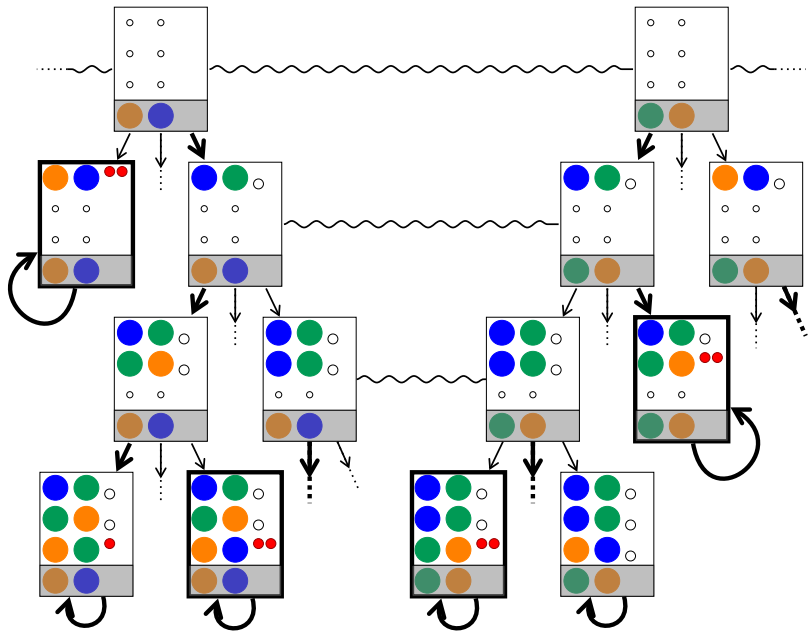
# The naive approach



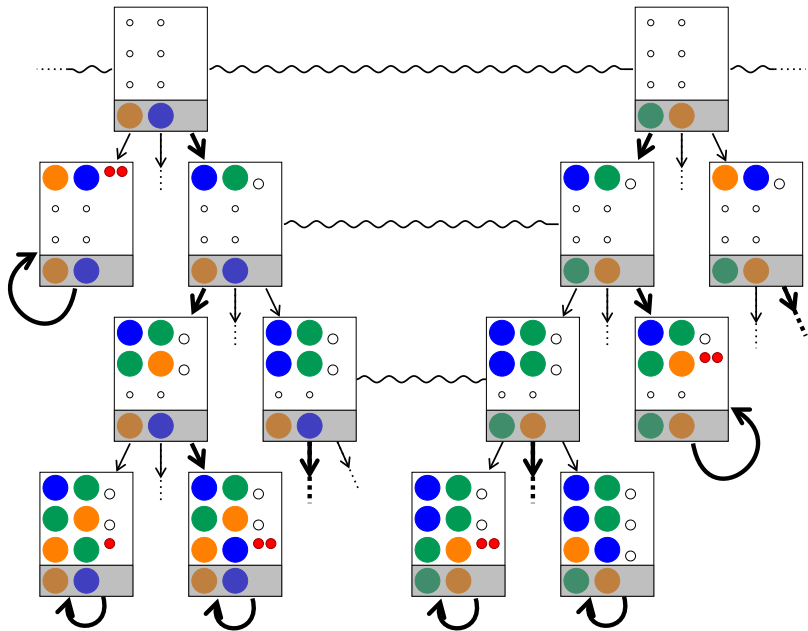
# The naive approach



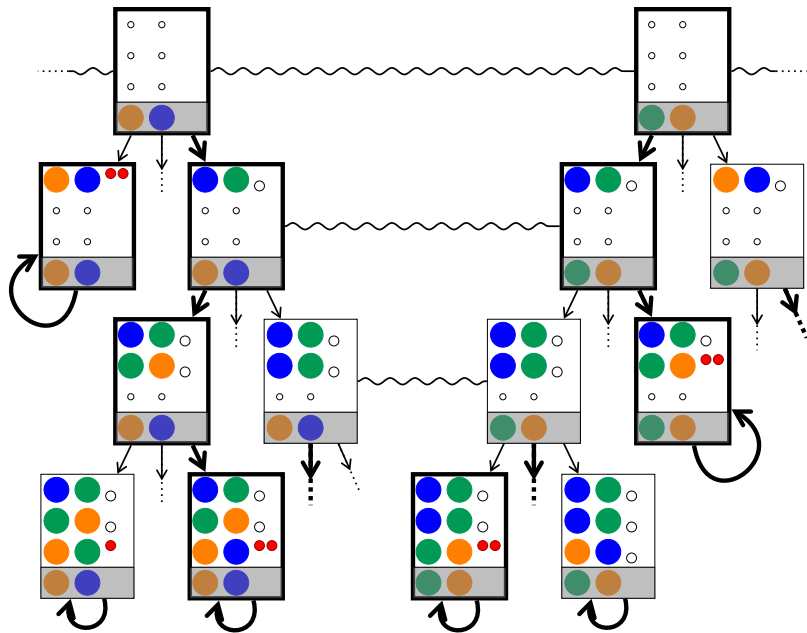
## The naive approach



## The naive approach



# The naive approach





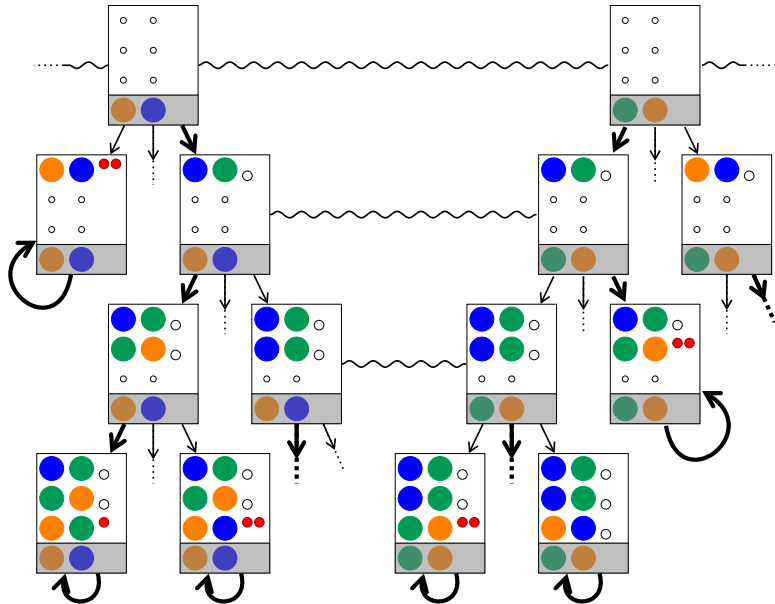
The naive approach is inefficient

The simplified Mastermind has  $7 \times 10^{112}$  **uniform strategies**

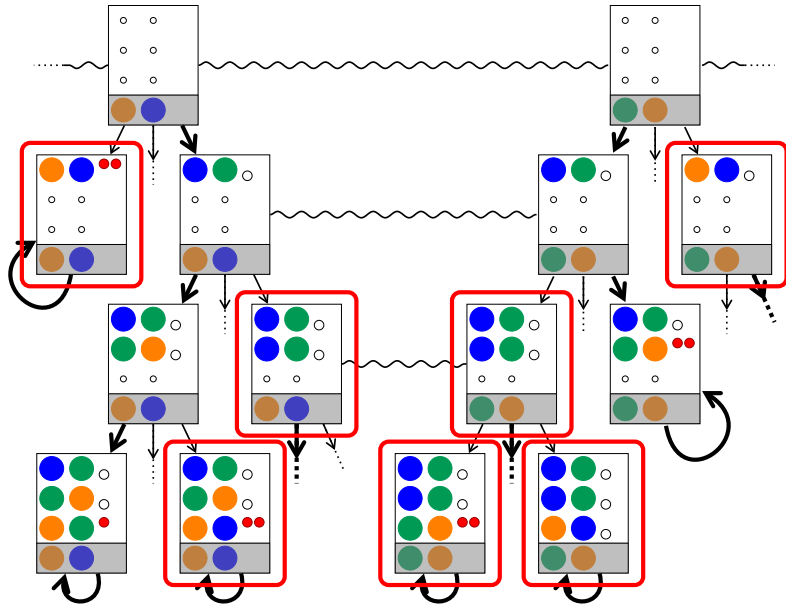
We need ways to **reduce** this number

$\Rightarrow$  **partial strategies**

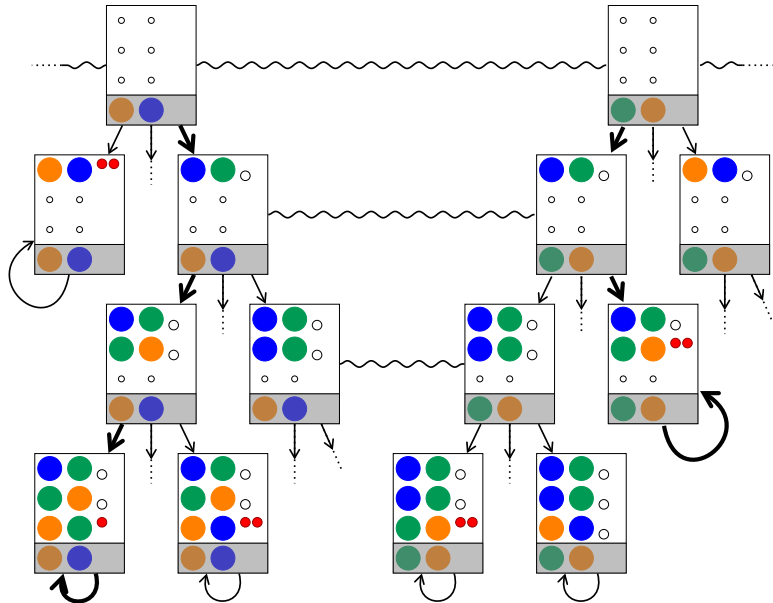
# Partial strategies



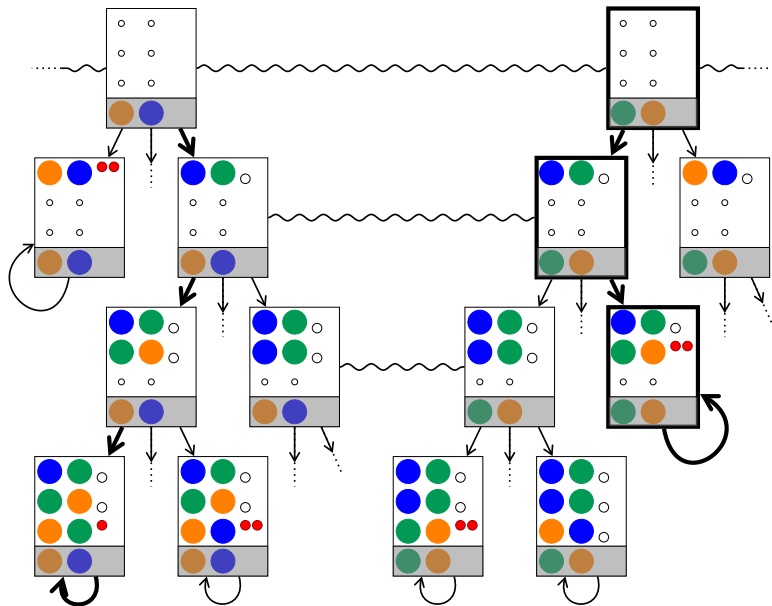
# Partial strategies



# Partial strategies

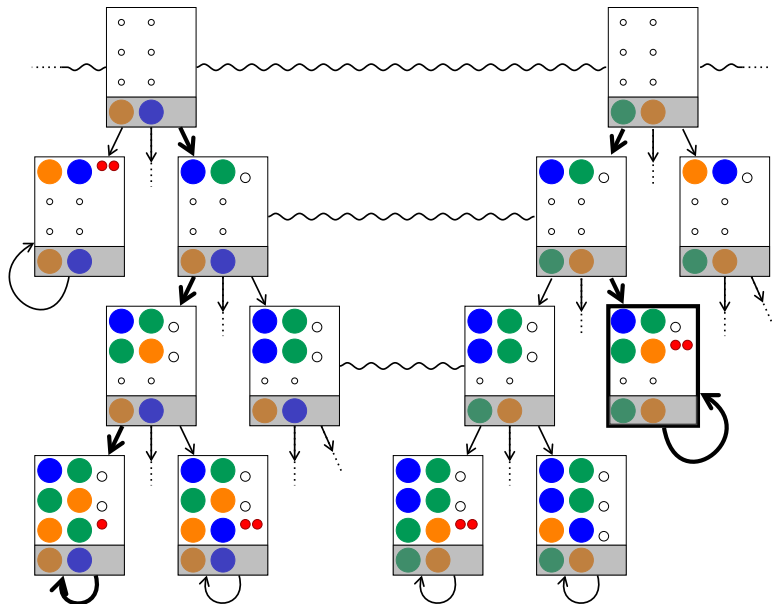


# Partial strategies



# Partial strategies

From  $7 \times 10^{112}$  to  $2 \times 10^6$  strategies



# The partial approach

To compute the states of  $Q'$  for which there exists a winning uniform strategy for some objective

1. generate each **partial strategy**  $f_{\Gamma}$
2. compute the states of  $Q'$  for which the **strategy is winning** with the corresponding *filter* algorithm
3. keep the states of  $Q'$  for which the strategy is winning for **all indistinguishable states**

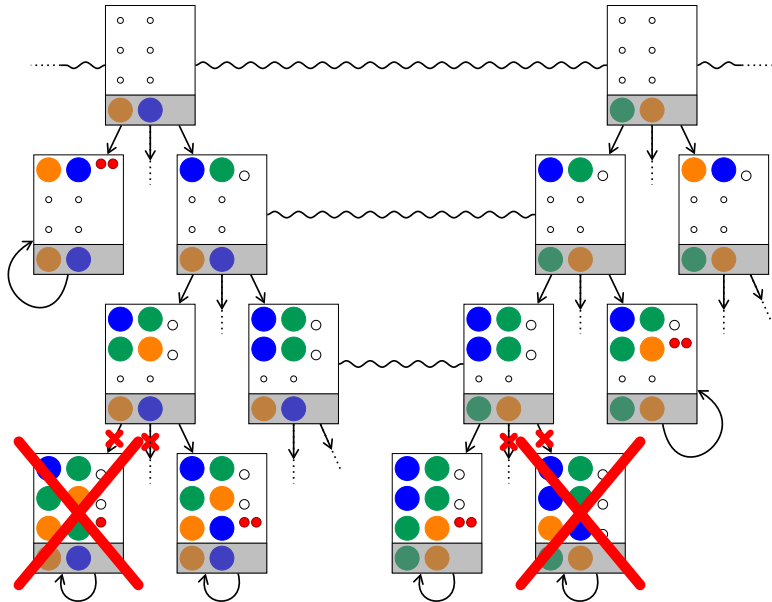
## Pre-filtering surely losing moves

1. It is **easy** to compute the moves belonging to a winning **general** strategy (PTIME)
2. If some move does not belong to a winning general strategy, it does not belong to a winning **uniform** one

⇒ We can **remove the losing moves** before enumerating the uniform strategies



# Pre-filtering surely losing moves



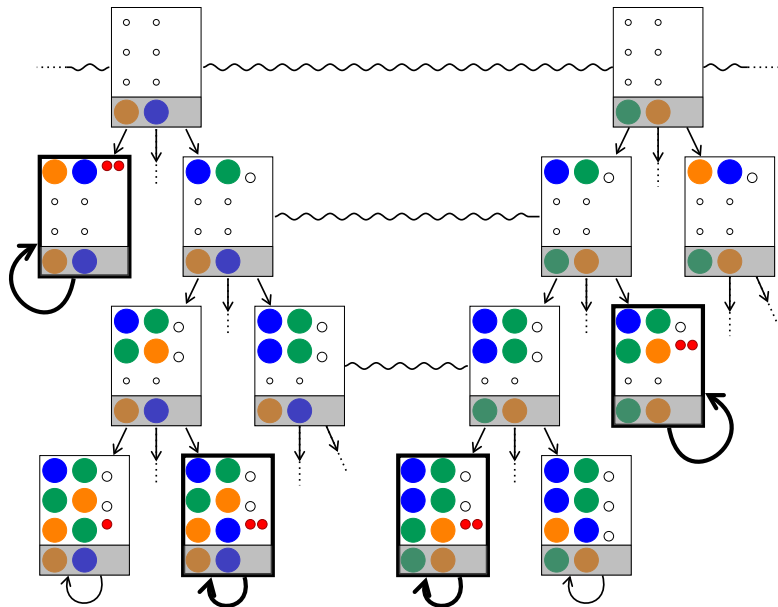
# Pre-filtering can help

Pre-filtering can drastically reduce the number of strategies:

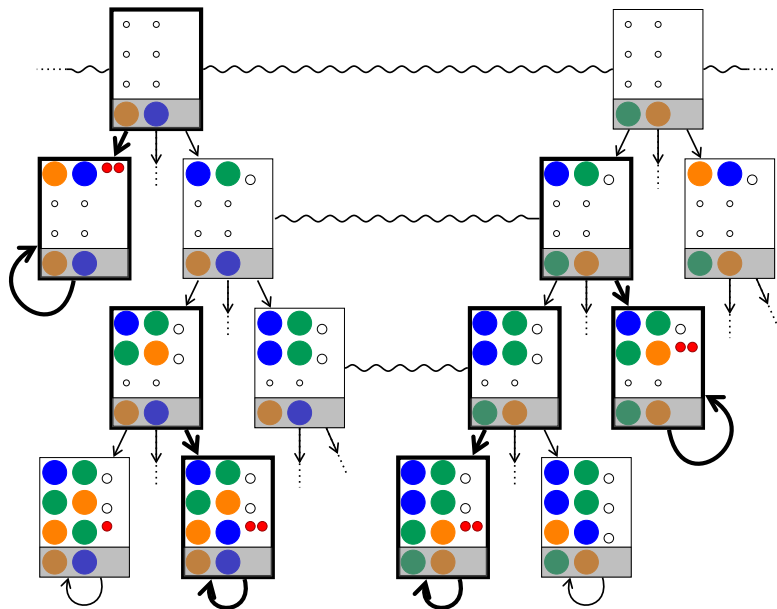
Naive approach: from  $7 \times 10^{112}$  to  $10^{22}$  uniform strategies

Partial approach: from  $2 \times 10^6$  to 2304 uniform partial strategies

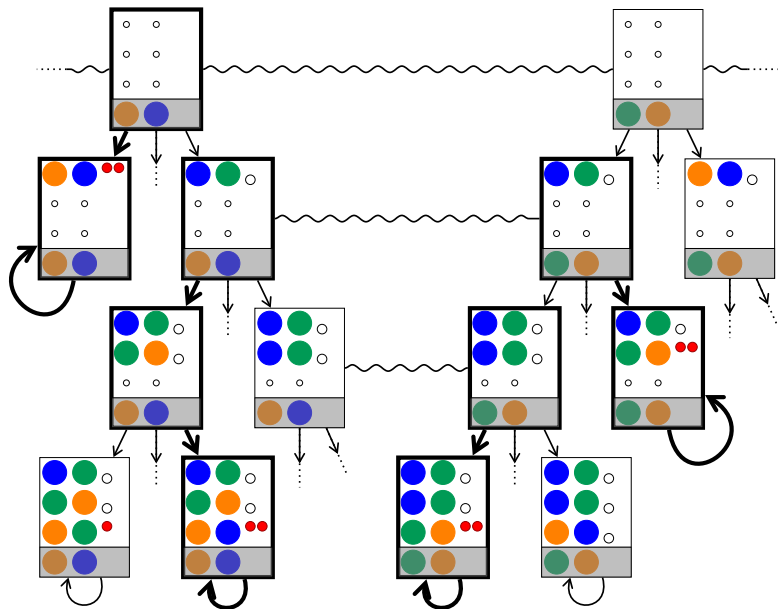
# The backward approach



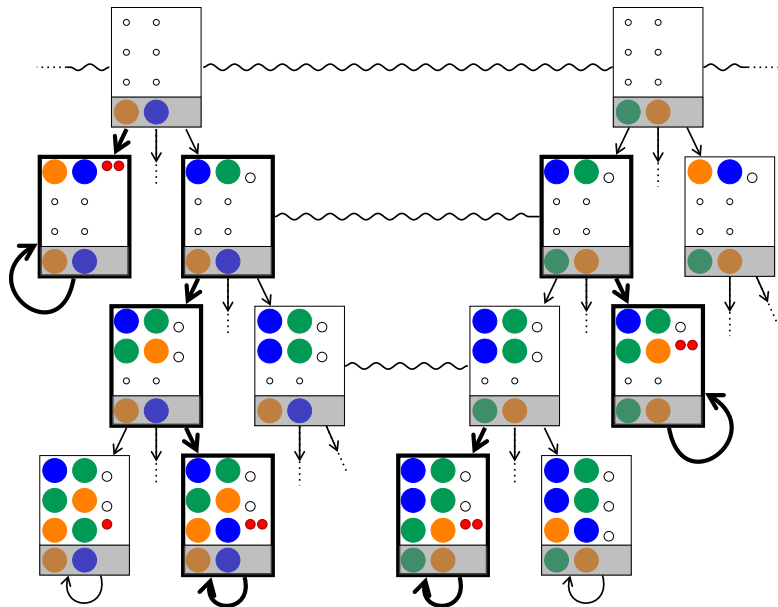
# The backward approach



# The backward approach



# The backward approach



# The backward approach



Build (parts of) winning strategies from the ground up

⇒ works for **reachability objectives**  
(e.g. reach a winning state)

⇒ does not work for **safety objectives**  
(e.g. avoid some losing state forever)

# Outline

## **Model checking uniform strategies**

Uniform strategies

Model checking approaches

### **Comparison with existing approaches**

Conclusion on model checking uniform strategies

Rich explanations for multi-modal logics

Conclusion



# Comparison with other approaches

Experimentally compared the three approaches with two existing ones:

1. Pilecki et al.
2. Huang and van der Meyden

Enriched with pre-filtering

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Experimentally compared the three approaches with two existing ones:

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Enriched with pre-filtering

Tested on 3 models, 6 properties

- ⇒ the naive approach is inefficient
- ⇒ pre-filtering sometimes helps
- ⇒ no general winner,  
different approaches are better in different situations

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- Conclusion on model checking uniform strategies**

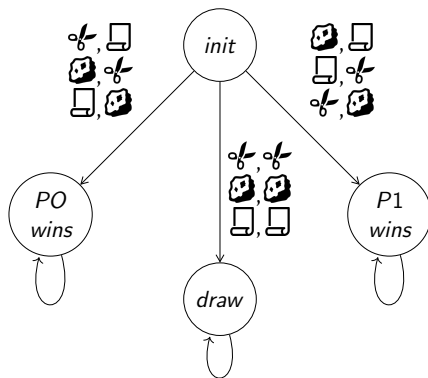
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Conclusion

# Model checking uniform strategies: more than winning a game

Coalitions: reason about strategies of **multiple agents**

Concurrent models: agents play **at the same time**



# Model checking uniform strategies: more than winning a game



Unconditional fairness constraints:

the player assumes the dealer is **fair**

i.e. if played infinitely often,  
all cards a given infinitely often

- ⇒ logic to reason  
about uniform strategies  
under fairness constraints
- ⇒ more complicated  
fixpoint computations

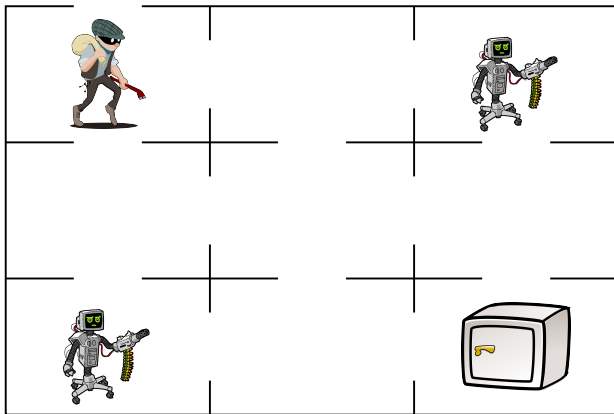


# Thesis contributions

- A logic to reason about uniform strategies under fairness constraints
- Three techniques to check uniform strategies (naive, partial and backward approaches)
- Pre-filtering surely losing moves (+ application to the approaches)
- An implementation of these approaches with PyNuSMV
- An experimental comparison with existing approaches

# Applications

⇒ Security policies



⇒ Networks:

strategies of machines to share data through unreliable links

# Outline

- Model checking uniform strategies

  - Uniform strategies

  - Model checking approaches

  - Comparison with existing approaches

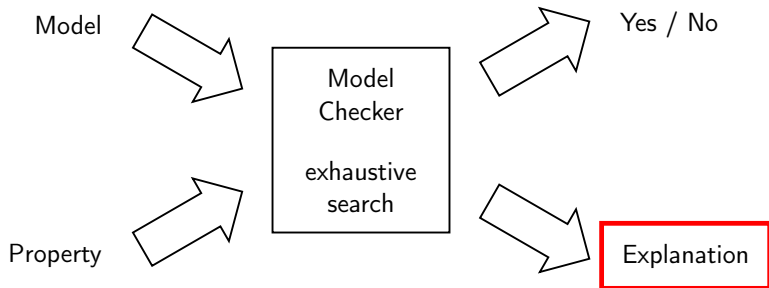
  - Conclusion on model checking uniform strategies

- Rich explanations for multi-modal logics**

- Conclusion



# Model checking can produce explanations



# Multi-modal logics have rich explanations

*"The player always eventually knows whether the first peg is blue"*

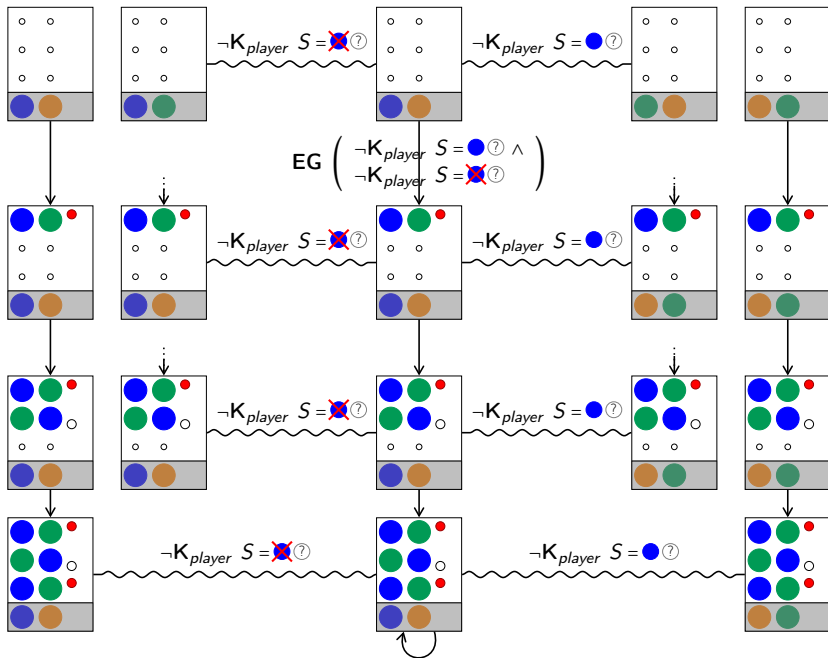
$$\mathbf{AF} (K_{player} S = \text{blue} \textcircled{?} \vee K_{player} S = \text{red} \textcircled{?})$$

Counter-example =

a part of the model showing why the property is violated

*"There is a play along which the player never knows whether the first peg is blue"*

$$\mathbf{EG} (\neg K_{player} S = \text{blue} \textcircled{?} \wedge \neg K_{player} S = \text{red} \textcircled{?})$$



# The problem

- Such explanations are difficult to generate and manipulate
- State-of-the-art model checkers return partial explanations



# Contribution

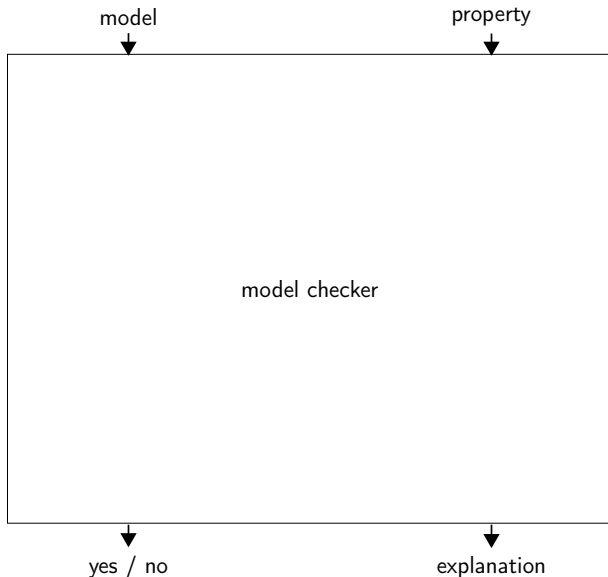
Many multi-modal logics can be translated into the mu-calculus:

(branching) time, knowledge, general strategies, etc.

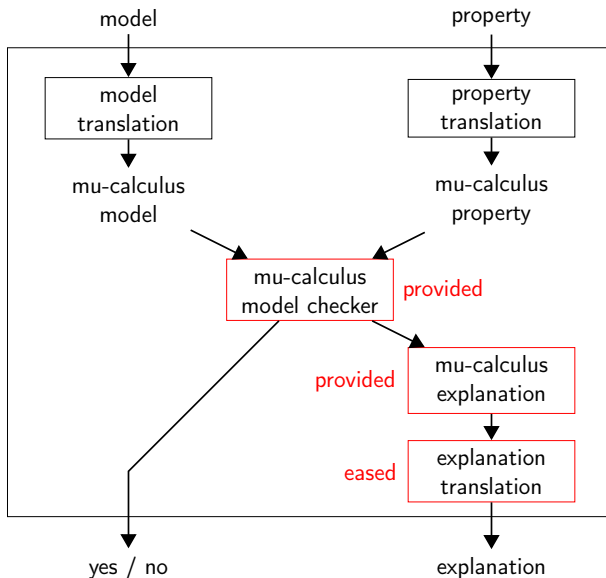
⇒ A mu-calculus-based model checking framework with rich explanations

(mu-calculus = a logic with modal and fixpoint operators)

# A mu-calculus based framework with rich explanations

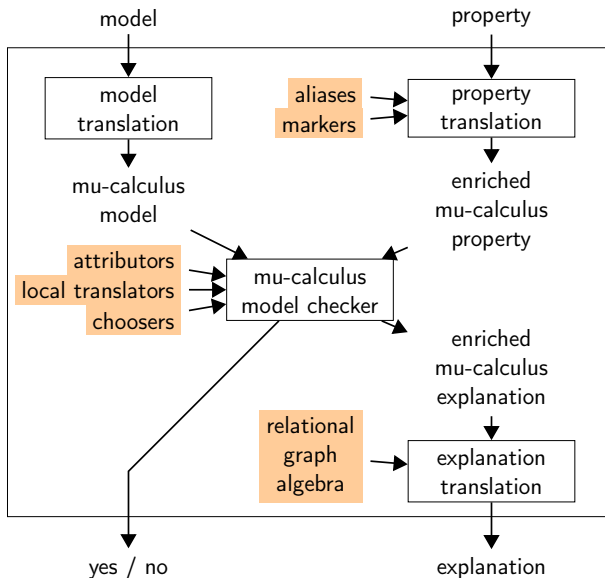


# A mu-calculus based framework with rich explanations





# A mu-calculus based framework with rich explanations



# Thesis contributions

- A mu-calculus based **model checker**...
- ...generating **rich explanations**...
- ...with features to **translate** them back into the original logic
  
- An **implementation** of the framework with PyNuSMV
  
- A **graphical tool** to visualize and manipulate the explanations

Applications to multi-modal logics: time, knowledge, strategies...

# Outline

Model checking uniform strategies

- Uniform strategies

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- Comparison with existing approaches

- Conclusion on model checking uniform strategies

Rich explanations for multi-modal logics

**Conclusion**

# Conclusion

Two main contributions:

1. techniques to model check uniform strategies under fairness constraints
2. a mu-calculus based framework with rich explanations and translation features

⇒ The framework could be used to manipulate the uniform strategies built by the model checking techniques

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