Reasoning about Strategies under Partial Observability and Fairness Constraints

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Running Example: A simple card game [1]

Three cards: A, K, Q
(A wins over K, K over Q, Q over A);

A player, a dealer.

[1] W. Jamroga, W. van der Hoek. *Agents that Know How to Play.* (2004)

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The dealer gives a card and keeps one;

the player can change his card
with the one on table.

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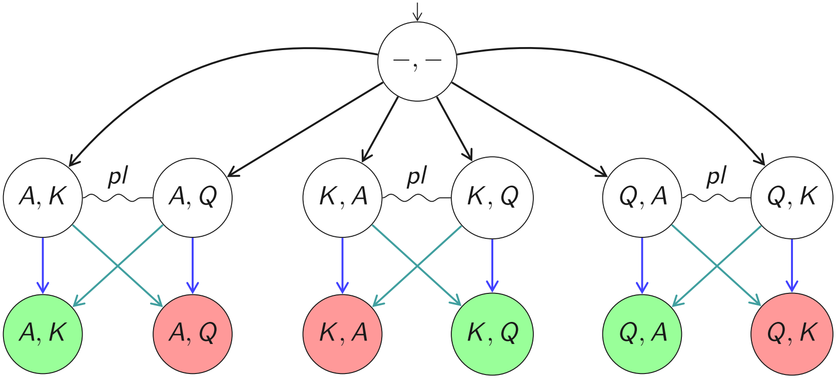
Running Example: A simple card game



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The dealer gives a card and keeps one;

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Variant: the player can play infinitely.

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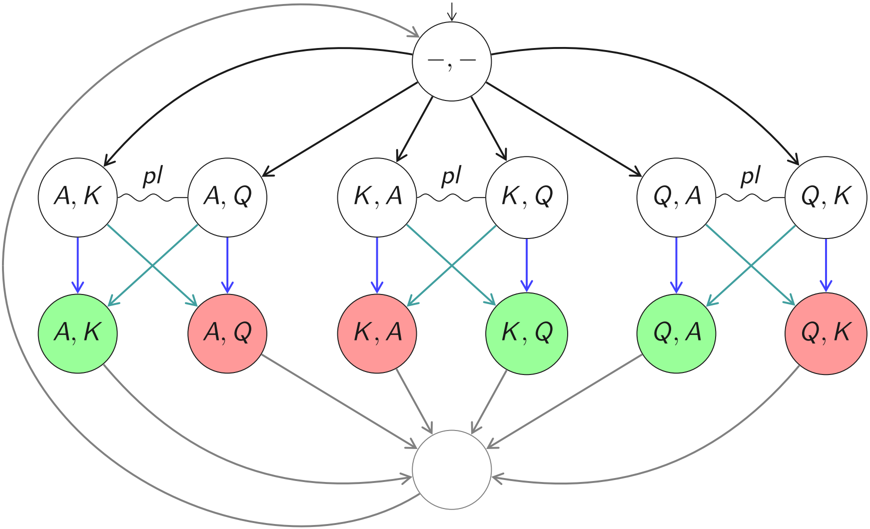

Reasoning about strategies

Model checking problem:
does the player have a strategy to win?

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$\Rightarrow$ it depends on the semantics!

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Model checking problem:
does the player have a strategy to win?

Under *ATL*, we consider all strategies.
The player has a strategy to win,
even if he cannot play it:
e.g., in $\langle A, K \rangle$, keep the card; in $\langle A, Q \rangle$, exchange it.

Reasoning about strategies

Model checking problem:
does the player have a strategy to win?

ATL: yes.

Under ATL_{ir}, we consider only memoryless uniform strategies.
There is no uniform strategy to win,
because the player cannot distinguish, e.g., $\langle A, K \rangle$ and $\langle A, Q \rangle$,
(winning actions are different in each case).

Reasoning about strategies

Model checking problem:
does the player have a strategy to win?

ATL: yes.

ATL_{ir}: no.

If we consider ATL_{ir} with a **fair dealer** and an **infinite play**,
the player can eventually win:
just use one uniform strategy, the right pair will finally come.

Reasoning about strategies

Model checking problem:
does the player have a strategy to win?

ATL: yes.

ATL_{ir}: no.

ATL_{ir} + fair dealer and infinite play: yes.

$\Rightarrow ATLK^F_{po}$: branching time, knowledge, memoryless uniform strategies and unconditional fairness constraints.

Outline

Strategies, Temporal Logics and Fairness

Strategies under Partial Observability and Fairness Constraints

Conclusion and Perspectives

ATL, reasoning about **strategies** of the agents. [2]

Syntax: Strategic modalities: $\langle\Gamma\rangle\mathbf{X}\ \phi$, $[\Gamma]\mathbf{G}\ \phi$, $\langle\Gamma\rangle[\phi_1\ \mathbf{U}\ \phi_2]$, etc.

Semantics: A state s satisfies $\langle\Gamma\rangle\ \pi$ iff there exists a set of **strategies** for agents in Γ such that **all enforced paths satisfy** π.

[2] Alur et al. *Alternating-time temporal logic.* (2002)

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Model checking:

$$[\![[\Gamma]\mathbf{G}\ \phi]\!] = \nu Z. [\![\phi]\!] \cap \mathit{Pre}_{[\Gamma]}(Z)$$

where $\mathit{Pre}_{[\Gamma]}(Z)$ is the set of states from which Γ cannot avoid to reach Z in one step.

[2] Alur et al. *Alternating-time temporal logic.* (2002)

ATL_{ir}, memoryless uniform strategies [3]

Only **memoryless uniform** strategies:

$$f_a : S \rightarrow Act \text{ such that } s \sim_a s' \implies f_a(s) = f_a(s')$$

Semantics: A state s satisfies $\langle\Gamma\rangle\ \pi$ iff there exists a set of **memoryless uniform** strategies for agents in Γ such that all paths enforced **from all** $s' \sim_\Gamma s$ satisfy π.

[3] Schobbens. *Alternating-time logic with imperfect recall*. (2004).

FairCTL: time and fairness constraints [4]

Add a set of **fairness constraints** $FC \subseteq 2^S$ to the model;
$\Rightarrow$ unconditional state-based fairness.

Only **fair paths** are considered:
$s \models \mathbf{E}\ \pi$ iff there exists a **fair** path from s satisfying π;
$s \models \mathbf{A}\ \pi$ iff all **fair** paths from s satisfy π.

[4] Clarke, Grumberg, Peled. *Model checking*. (2000).

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Model checking:

$$[\![\mathbf{EG}\ \phi]\!] = \nu Z.[\![\phi]\!] \cap \bigcap_{fc \in FC} Pre(\mu Y.(Z \cap fc) \cup ([\![\phi]\!] \cap Pre(Y)))$$

where $Pre(Z)$ is the set of states having a successor in Z.

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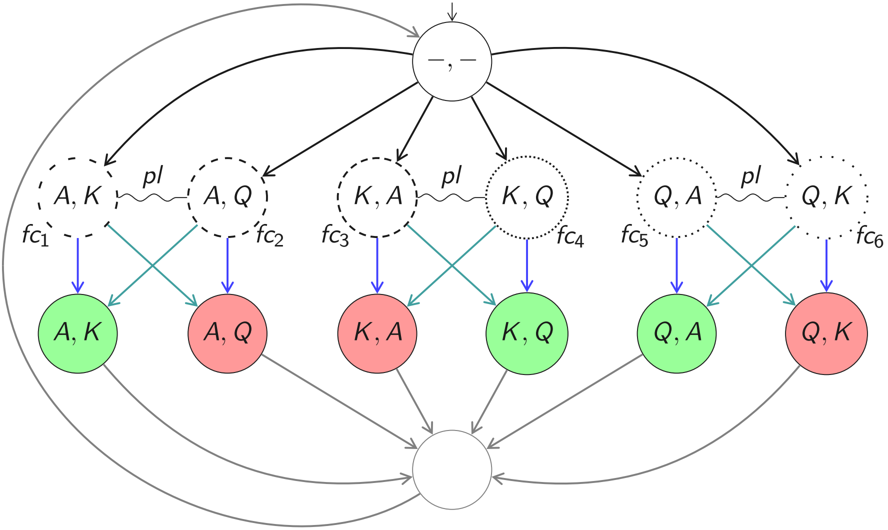
Adding fairness constraints to the card game



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$ATLK_{po}^{F}$ = *FairCTL*, knowledge and ATL_{ir} with fairness

Syntax: CTL (**EX**, **AG**, etc.), knowledge ($\mathbf{K}_{ag}$, $\mathbf{C}_{g}$, etc.) and strategies ($\langle\Gamma\rangle\mathbf{F}$, $[\Gamma]\mathbf{U}$, etc.)

Semantics: A state s satisfies $\langle\Gamma\rangle\ \pi$ iff there exists a set of **memoryless uniform** strategies for agents in Γ such that all **fair** paths enforced **from all** $s' \sim_{\Gamma} s$ satisfy π.

To model check $ATLK^F_{po}$, we defined $ATLK^F_{fo}$ and its model checking

$ATLK^F_{fo} = FairCTL$ + knowledge + ATL with fairness

$ATLK^F_{fo}$ **semantics:** A state s satisfies $\langle\Gamma\rangle\ \pi$ iff there exists a set of memoryless strategies (not necessarily uniform) for agents in Γ such that all **fair** paths enforced (from s only) satisfy π.

To model check $ATLK^F_{po}$, we defined $ATLK^F_{fo}$ and its model checking

$ATLK^F_{fo} = \mathit{FairCTL}$ + knowledge + ATL with fairness

$ATLK^F_{fo}$ **semantics:** A state s satisfies $\langle\Gamma\rangle\ \pi$ iff there exists a set of memoryless strategies (not necessarily uniform) for agents in Γ such that all **fair** paths enforced (from s only) satisfy π.

$ATLK^F_{fo}$ **model checking:**

$$[\![[\Gamma] G\phi]\!]^F_{fo} = \nu Z.[\![\phi]\!]^F_{fo} \cap \bigcap_{fc \in FC} Pre_{[\Gamma]}(\mu Y.(Z \cap fc) \cup ([\![\phi]\!]^F_{fo} \cap Pre_{[\Gamma]}(Y)))$$

$ATLK_{po}^F$ model checking

A state s satisfies $\langle \Gamma \rangle\ \pi$ iff there exists a set of **memoryless uniform strategies** for agents in Γ which allows Γ to enforce π in all **states indistinguishable from** s, considering only **fair paths**.

$ATLK_{po}^{F}$ model checking

A state s satisfies $\langle\Gamma\rangle\ \pi$ iff there exists a set of **memoryless uniform strategies** for agents in Γ which allows Γ to enforce π in all **states indistinguishable from** s, considering only **fair paths**.

To get all the states satisfying $\langle\Gamma\rangle\ \pi$:

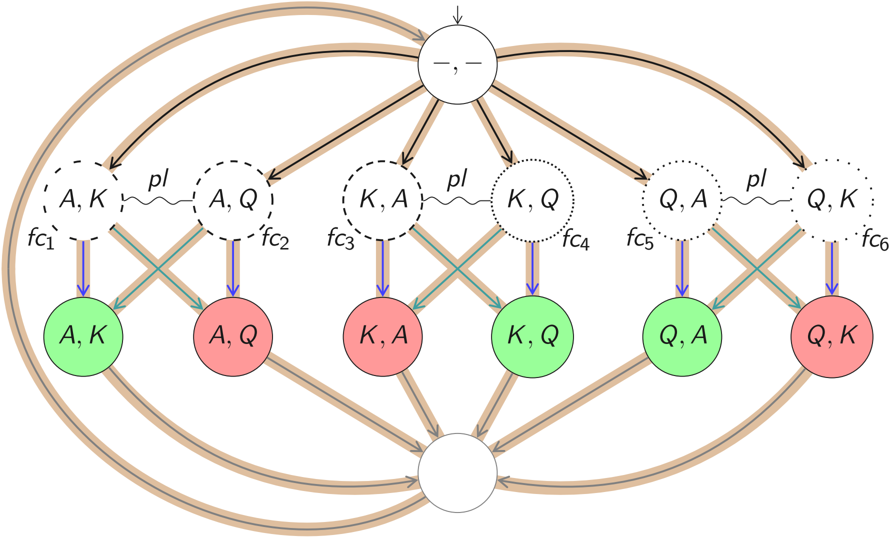
1. List all the memoryless uniform strategies;
2. Use $ATLK_{fo}^{F}$ model checking to get states satisfying the property **in this strategy**;
3. Then restrict to set of undistinguishable states.

$ATLK_{po}^{F}$ model checking: *Split* algorithm

Split the state/action pairs into memoryless uniform strategies.

1. Get all conflicting equivalence classes;
2. If there are none, the set is itself a memoryless uniform strategy.
3. Otherwise, choose a conflicting equivalence class;
4. Split it;
5. and recursively call *Split* on the rest.

$ATLK_{po}^F$ model checking example: $\langle player \rangle \mathbf{F}$ win



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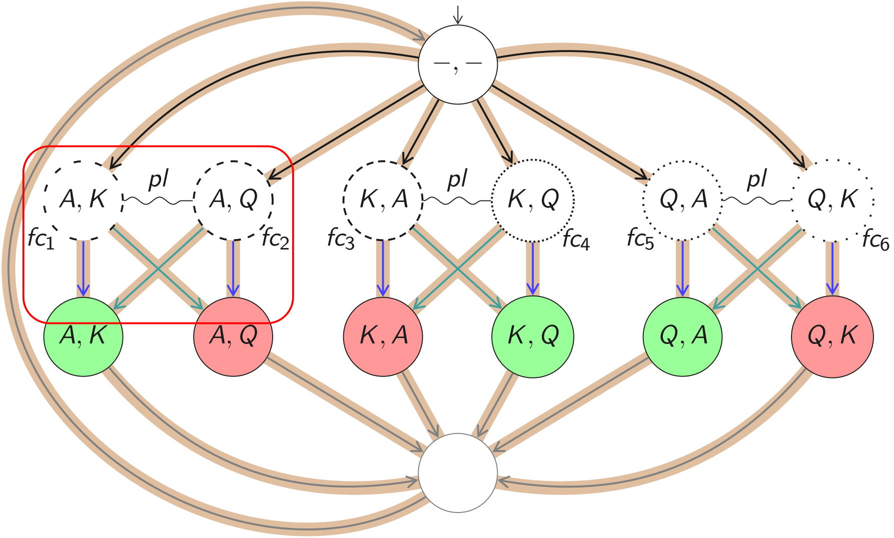
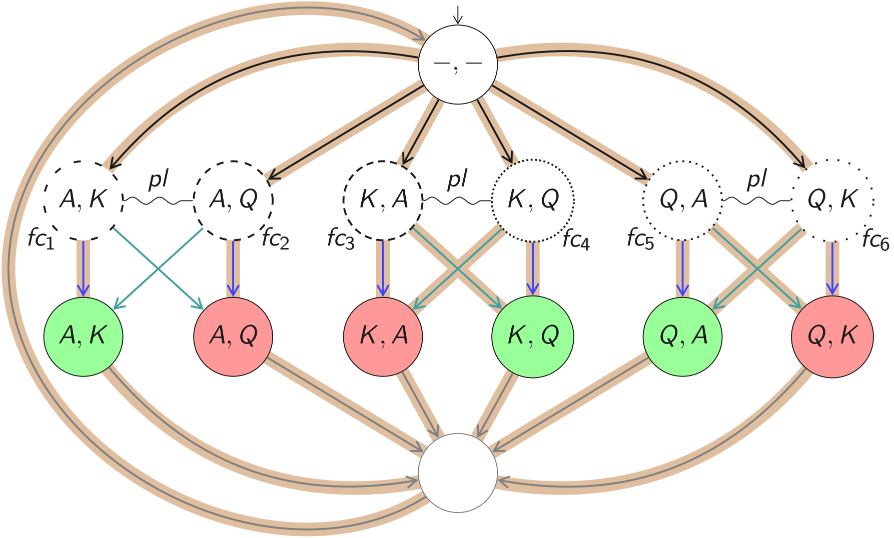


$ATLK^F_{po}$ model checking example: $\langle player \rangle \mathbf{F}$ win



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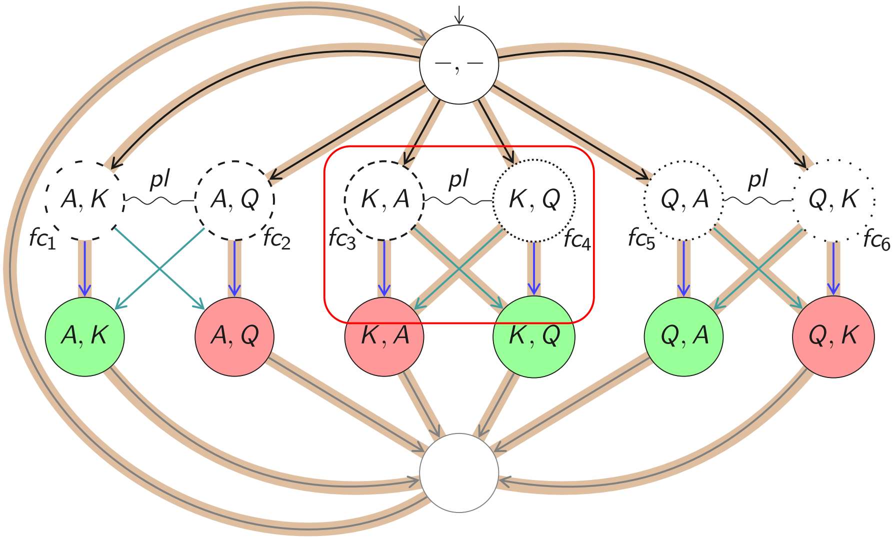

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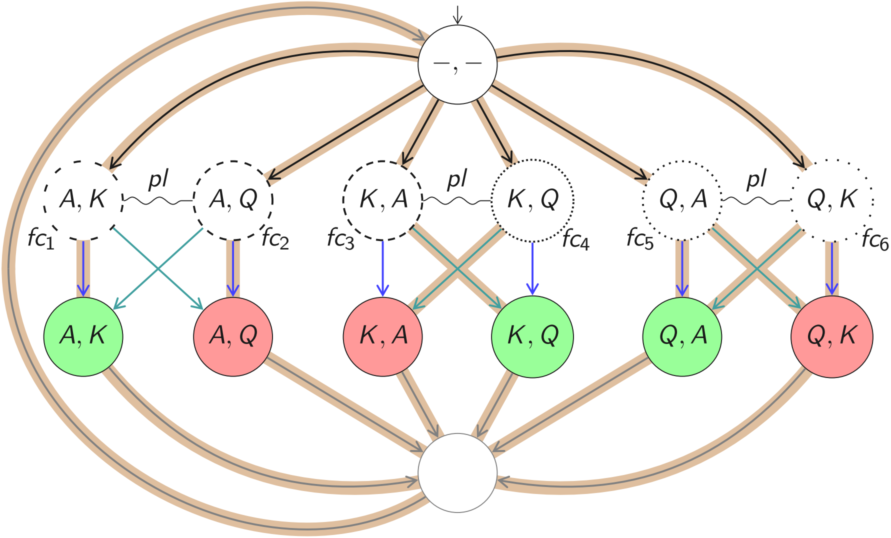
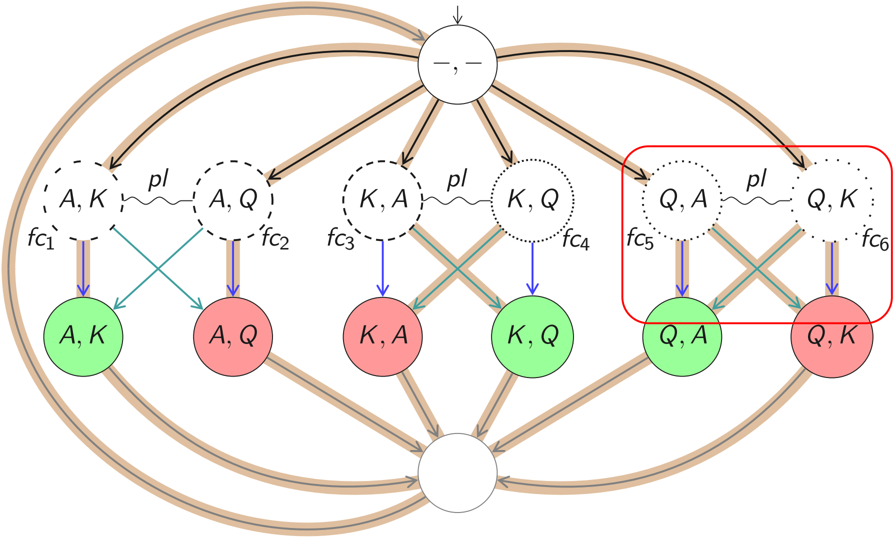


$ATLK_{po}^{F}$ model checking example: $\langle player \rangle \mathbf{F}$ win



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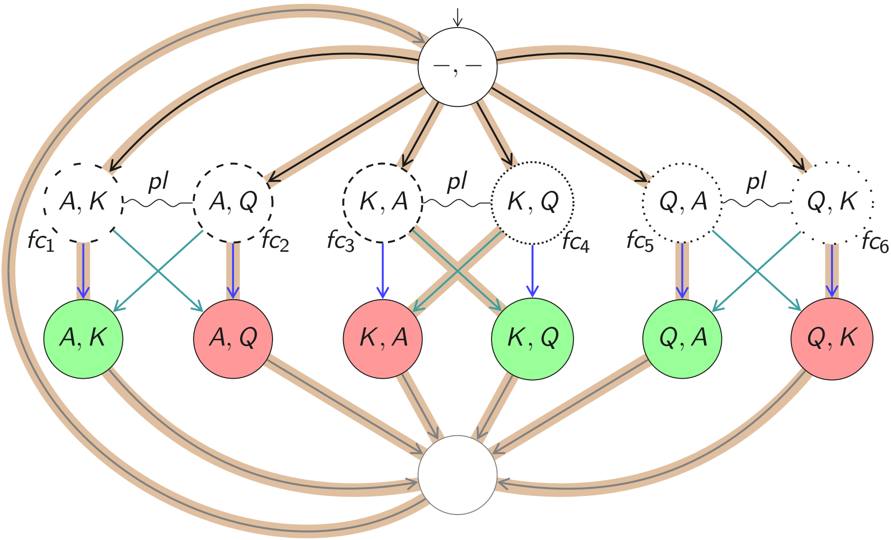

Improving the algorithm: alternating between filtering states and splitting strategies

We can alternate between filtering states that belong to a strategy, and splitting non-uniform strategies into uniform ones.

The filtering is correct since $s \not\models^{F}_{fo} \langle\Gamma\rangle\ \pi \implies s \not\models^{F}_{po} \langle\Gamma\rangle\ \pi$.

1. Filter current sub-graph for getting states with a strategy;
2. Split on one conflicting equivalence class (if any; otherwise, stop);
3. call the algorithm again with each split sub-graph.

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Conclusion

$ATLK_{po}^{F}$: branching time, knowledge and strategies under partial observability and (unconditional state-based) fairness constraints.

(Symbolic) model checking algorithm based on $ATLK_{fo}^{F}$ **model checking** and **splitting the graph** into memoryless uniform strategies.

Future work

Develop counter-examples for $ATLK_{po}^{F}$
(for model understanding, controller synthesis)

Implement a model checker for $ATLK_{po}^{F}$
with counter-examples generation
(with PyNuSMV, a new Python framework based on NuSMV [5])

[5] S. Busard, C. Pecheur. *PyNuSMV: NuSMV as a Python Library.* (2013)

Thank you.
Questions?