Backward Model Checking of Uniform Strategies

A Backward-traversal-based Approach for Symbolic Model Checking of Uniform Strategies for Constrained Reachability

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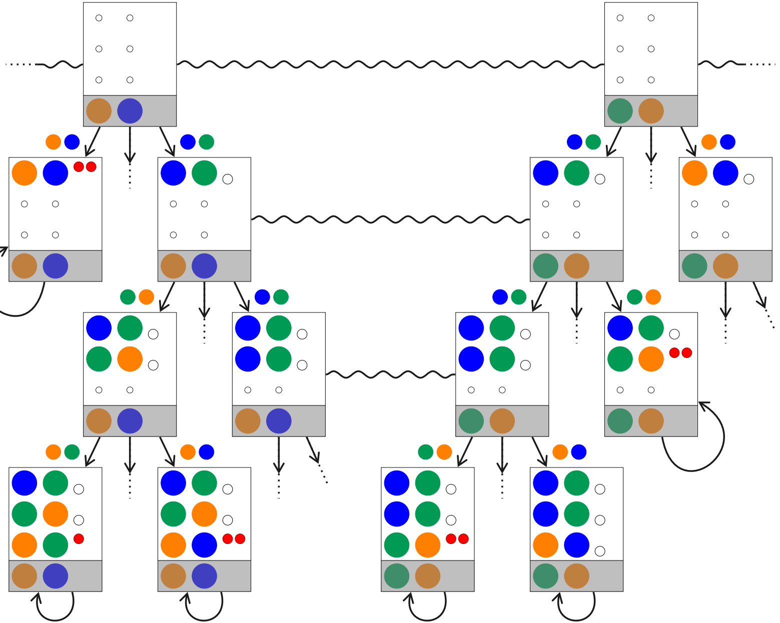
Running example: Mastermind

Simplified Mastermind: 3 colors, 3 turns, 2 pegs (different colors)

Simplified Mastermind FSM (1122 states)



Mastermind

Is there a **strategy**
for the guesser to find the solution?

A **uniform** strategy!

ATL_{ir}, ATL with imperfect information and recall

$ATL_{ir} =$ logic for **uniform** strategies and CTL-**like objectives** in concurrent multi-agent systems

"the guesser has a strategy to eventually win the game"

$\langle\langle guesser \rangle\rangle \mathbf{F}\ win$

"the guesser has a strategy to never play a blue peg until winning"

$\langle\langle guesser \rangle\rangle [\neg blue\ peg\ \mathbf{U}\ win]$

"the guesser has a strategy to never lose"

$\langle\langle guesser \rangle\rangle \mathbf{G}\ \neg lose$

⚠ Model checking ATL formulas is **easy** (PTIME), but model checking ATL_{ir} formulas is Δ_P^2-**complete**!

Existing approaches for ATL_{ir} model checking

The **partial** approach:
S. Busard, C. Pecheur, H. Qu, F. Raimondi (2014)
Improving the Model Checking of Strategies under Partial Observability and Fairness Constraints

The **early** approach:
J. Pilecki, M.A. Bednarczyk, W. Jamroga (2014)
Synthesis and Verification of Uniform Strategies for Multi-agent Systems

The **symbolic** approach:
X. Huang, R. van der Meyden (2014)
Symbolic Model Checking Epistemic Strategy Logic

The partial approach

1. generate all **partial strategies** from states that matter (through a **forward** traversal)

2. check each strategy against the objective

+ early termination

The early approach

- We do not need to get a completely determined adequate partial strategy before checking it
- We can stop if all extensions of the current strategy are winning
- We can stop if **no** general extension is winning

$\Rightarrow$ alternate between extending a partial strategy and checking whether all or no extensions are winning

+ early termination

The symbolic approach

1. Encode the uniform strategies in the **states** of a derived model
2. Perform fixpoint computations on the derived model

$\Rightarrow$ Compute **all** winning strategies at the same time (symbolically)

$\Rightarrow$ explosion of the number of derived states

Pre-filtering surely losing moves

1. It is **easy** to compute the moves belonging to a winning **general** strategy (PTIME)

2. If some move does not belong to a winning general strategy, it does not belong to a winning **uniform** one

$\Rightarrow$ We can **remove the losing moves** before enumerating or encoding the uniform strategies

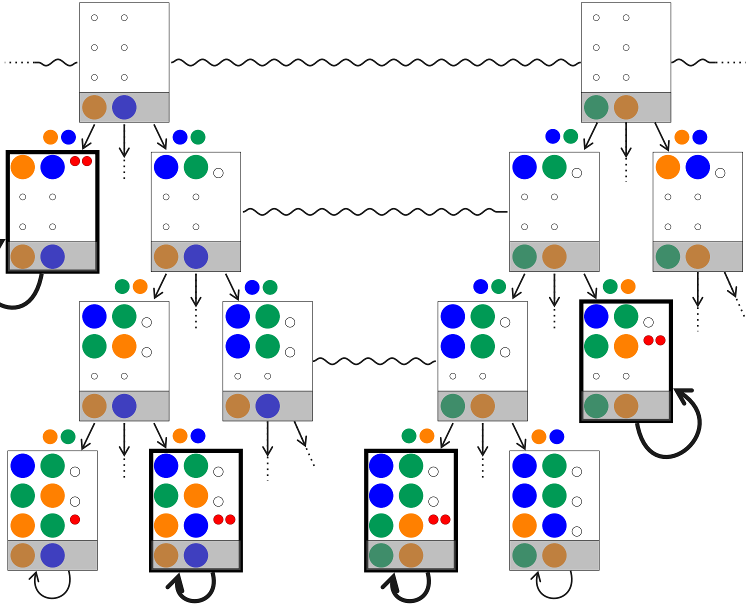
Objective: a new approach

Partial and early approaches:
forward traversal to generate strategies
+ **backward** traversal to check strategies

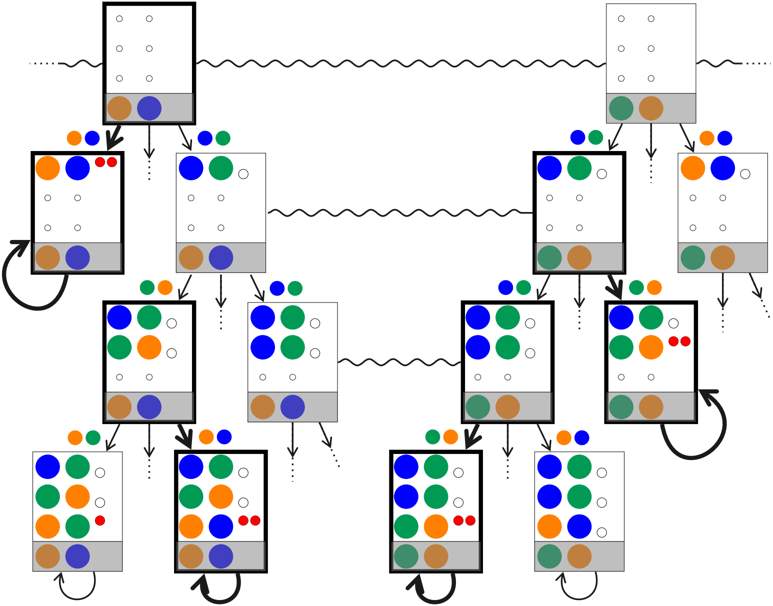
⇒ Design an approach working with a **backward** traversal only
⇒ The backward approach

Outline

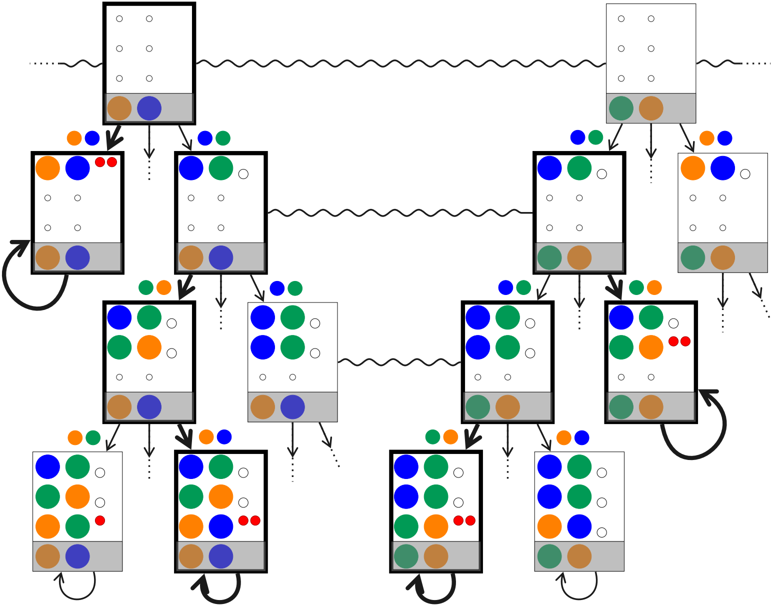
The backward approach



The backward approach



The backward approach



The backward approach

For $\langle\!\langle \Gamma \rangle\!\rangle[P_1 \mathbf{U} P_2]$,

1. start with the moves in states satisfying P_2
2. split them into non-conflicting subsets M_Γ
3. then iterate:
    - compute states for which the current strategy M_Γ is surely winning or surely losing
    - get the **compatible** moves in states satisfying P_1 and reaching states of M_Γ
    - split the newly discovered moves into **non-conflicting subsets** and **extend** M_Γ with them

The backward approach

For $\langle\langle\Gamma\rangle\rangle[P_1 \; \mathbf{U} \; P_2]$,

1. start with the moves in states satisfying P_2
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Limitations

- Limited to **constrained reachability** objectives:
  $\langle\!\langle\Gamma\rangle\!\rangle\mathbf{X}$, $\langle\!\langle\Gamma\rangle\!\rangle\mathbf{F}$, $\langle\!\langle\Gamma\rangle\!\rangle\mathbf{U}$ operators (and their $[\![\Gamma]\!]$ dual),
  Cannot handle $\langle\!\langle\Gamma\rangle\!\rangle\mathbf{G}$ and $\langle\!\langle\Gamma\rangle\!\rangle\mathbf{W}$ operators

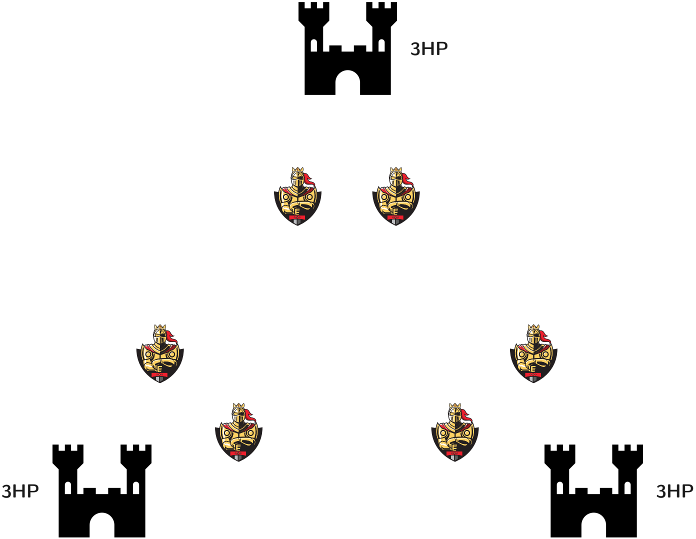
  But can be **mixed** with other approaches to handle other operators

- Split new moves into non-conflicting non-maximal subsets
  $\Rightarrow$ doubly exponential!

  But experiments show competitive results

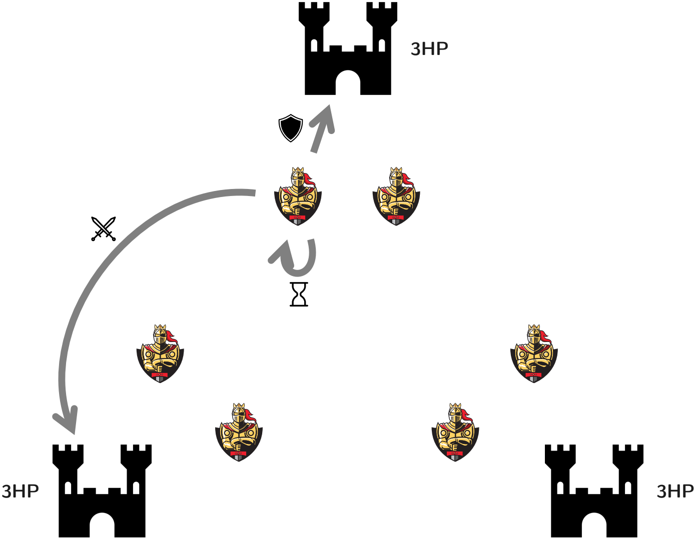
Outline

The three-castle model [1]



[1] J. Pilecki, M.A. Bednarczyk, W. Jamroga (2014).
Synthesis and Verification of Uniform Strategies for Multi-agent Systems

The three-castle model [1]   (±80K strategies per soldier)



[1] J. Pilecki, M.A. Bednarczyk, W. Jamroga (2014).
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Tested formulas

"the two first castles can defeat the third one"

$$\langle\!\langle Castle_1, Castle_2 \rangle\!\rangle \mathbf{F}\ Castle_3\ defeated$$

Satisfied by all tested instances

"Soldier 1 and Soldier 2 can defeat all castles"

$$\langle\!\langle Soldier_1, Soldier_2 \rangle\!\rangle \mathbf{F}\ all\ defeated$$

Violated by all tested instances

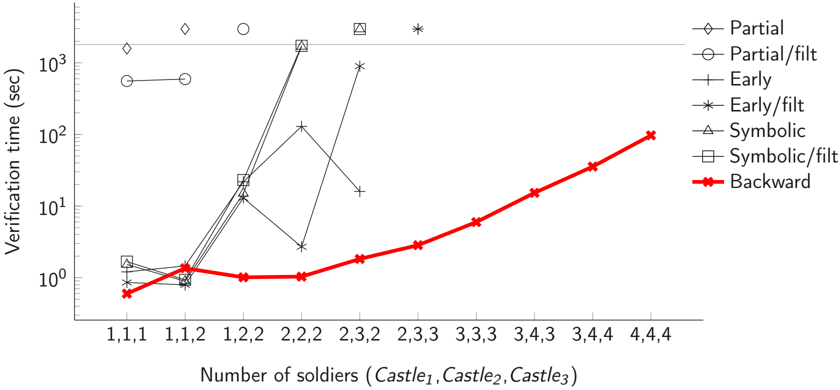
Test setting

- Approaches implemented in the same BDD-based framework (with PyNuSMV, a Python framework based on NuSMV)
- Run on instances of increasing size
- Each test run with a 1800 seconds time limit

Results

"the two first castles can defeat the third one"

$\langle\langle Castle_1, Castle_2 \rangle\rangle \mathbf{F}\ Castle_3\ defeated$



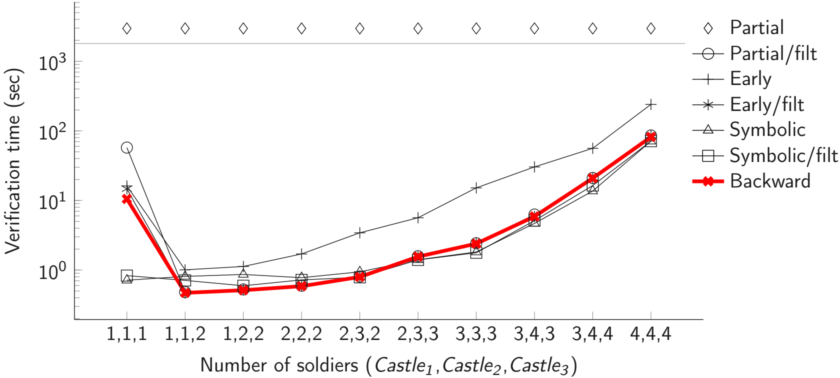
Analysis

- partial approaches have difficulties finding a winning strategy
  pre-filtering helps
- symbolic approaches are better
  pre-filtering does nothing
- early approaches are better
  with irregularities
- backward approach works well
  focus on reaching the target states

Results

"Soldier 1 and Soldier 2 can defeat all castles"

$\langle\langle Soldier_1, Soldier_2 \rangle\rangle \mathbf{F}\ all\ defeated$



Analysis

- partial approach has too many strategies to check
- pre-filtering solves the problem directly
- early approach concludes a bit slower
- backward approach analyses one strategy before concluding

Results summary

The backward approach is similar or better than the other approaches **on the tested model**

because it is **goal-driven** and can **rule losing strategies out**

Outline

Conclusion

The backward approach for finding uniform winning strategies:

- build uniform strategies from the target states
- check that extensions are already winning or surely losing

⚠ Limited to constrained reachability objectives ($\langle\langle\Gamma\rangle\rangle$**U** operator)

Compared with existing approaches **on one model**:

- works better than the other approaches in the first case,
- works similarly to the other approaches in the second

Future work

- Experiment on other case studies
- Experiment with mixed approaches

Thank you!

Questions?